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Graph Theory

Defn:- Graph is a mathematical representation of a network and it describes the relationship between lines and points. A graph consists of some points & lines between them. The length of the lines and position of the points do not matter. Each object in a graph is called a node

A graph 'G' is a set of vertex, called nodes 'V' which are connected by edges, called links 'e'. Thus G = (V, e).

Vertex (Node):- A node v is an intersection point of a graph. It denotes a location such that as a city, a road intersection, or a transport terminal likes stations, harbours and airports.

Edge (Link):- An edge e is a link between two nodes. A link denotes movements between nodes. It has a direction that is generally represented as an arrow. If an arrow is not used, it means the link is ~~bidirectional~~ bidirectional.

Transport geography can be defined by a graph. Most networks, namely road, transit, and rail networks, are defined more by their links than by nodes. But it is not true for all transportation networks. For instance, air networks are defined more by their nodes than by their links

Since links are mostly not clearly defined. A communication (tele) can be @ Systemed also be represented as a network. Mobile telephone networks or the internet is the considered the most complex graph. However, cell phones and antennas can be represented as nodes whereas links could be individual phone calls. The cores of the internet or servers can also be represented as nodes while the physical infrastructure between them, like ~~fiber~~ optic cables, can act as links. This suggests that all tranport networks can be represented by graph theory in some way.

A graph $G_2 = (V, E)$ consists of two sets V and E. The elements of V are called the vertices and the elements of E are called the edges of G. Each edge is a pair of vertices. For instance, the sets $V = \{1, 2, 3, 4, 5\}$ and $E = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}\}$ define a graph with 5 vertices and 4 edges.

Graphs have natural visual representations in which each vertex is represented by a point and each edge by a line connecting two points.

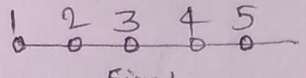


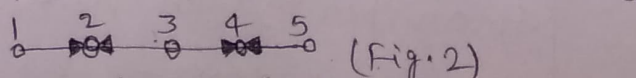
Fig. 1

Fig 1 : Graph $G_2 = (V, E)$ with $V = \{1,2,3,4,5\}$ and $E = \{\{1,2\},\{2,3\},\{3,4\},\{4,5\}\}$

By altering the definition, we can obtain different types of graphs. For instance,

① by replacing the set E with a set of ordered pairs of vertices, we obtain a directed graph ~~(or)~~ or digraph, also known as oriented graph or ographh. Each edge of that graph has a specific orientation indicated in the diagram representation by an arrow (Fig. 2)

Observe that in general two vertices i and j of an oriented graph can be connected by two edges directed opposite to each other, i.e. (i, j) and (j, i)



(Fig. 2)

Fig. 2 : An oriented graph $G_1 = (V, E)$ with $V = \{1,2,3,4,5\}$ and $E = \{\{1,2\},\{2,3\},\{3,4\},\{4,5\}\}$.